

# The decays of on-shell and off-shell polarized gauge bosons into massive quark pairs at NLO QCD <sup>1</sup>

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## Abstract

We discuss the polar angle decay distribution in the decay of on-shell and off-shell polarized  $(W, Z)$  gauge bosons into massive quark pairs. In particular for the off-shell decays in  $H \rightarrow (W, Z) + (W^*, Z^*) (\rightarrow q_1 \bar{q}_2)$  it is important to keep the masses of the charm and bottom quarks at their finite values since the scale of the problem is not set by  $m_{W,Z}^2$  but by the off-shellness of the gauge boson which varies in the range  $(m_1 + m_2)^2 \leq q^2 \leq (m_H - m_{W,Z})^2$ .

## 1 Introduction

The polarization of  $W$  and  $Z$  bosons produced in electroweak processes is in general highly nontrivial. One therefore has a rich phenomenology of polarization effects in  $(W, Z)$  production and decay which will be explored in present and future high energy experiments. The polarization of the  $W$  and  $Z$  bosons can be probed by decay correlations involving the momenta of the final state leptons or quarks in the decays of the polarized  $(W, Z)$  bosons. Bottom and charm quark mass effects are moderate for on-shell decays but are important for off-shell decays in particular in the vicinity of the threshold where the off-shell value  $q^2$  becomes comparable to the quark masses.

## 2 Angular decay distribution

In the  $W$  rest frame the angular decay distribution of a polarized on-shell spin-1  $W$  boson into a pair of fermions is given by

$$W(\theta) \propto \frac{3}{2} \sum_{m,m'=0,\pm} \rho_{mm} d_{mm'}^1(\theta) d_{mm'}^1(\theta) H_{m'm'}$$

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<sup>1</sup>Invited talk given by J.G. Körner at the 20th International Symposium on Spin Physics, September 17-22, 2012, Dubna Russia. To be published in the Proceedings.

$$\begin{aligned}
&= \frac{3}{8} \cos^2 \theta (\rho_{++} - 2\rho_{00} + \rho_{--})(H_{++} - 2H_{00} + H_{--}) \\
&\quad + \frac{3}{4} \cos \theta (\rho_{++} - \rho_{--})(H_{++} - H_{--}) \\
&\quad + \frac{3}{8} ((\rho_{++} + 2\rho_{00} + \rho_{--})(H_{++} + 2H_{00} + H_{--}) - 4\rho_{00}H_{00}). \tag{1}
\end{aligned}$$

The  $\rho_{mm}$  are the process dependent (unnormalized) density matrix elements of the spin-1 gauge boson and the  $H_{mm}$  are the (unnormalized) universal polarized decay functions needed to analyze the polarization of the gauge boson. The polar angle  $\theta$  is the angle between the  $z$  direction defined by the production process and a  $z'$  direction specified by the decay process. When the  $z'$  axis is defined by the antiquark in the decay  $W^+ \rightarrow c\bar{b}$  the (unnormalized)  $O(\alpha_s)$  analyzing polarized decay functions  $H_{mm}$  are given by

$$\begin{aligned}
H_{++} &= 8N_c q^2 \left[ 1 + \frac{\alpha_s}{6\pi} \left( 1 + (\pi^2 + 16)m_2/\sqrt{q^2} \right) + \dots \right], \\
H_{00} &= 8N_c q^2 \left[ 0 + \frac{\alpha_s}{6\pi} \left( 4 - 2\pi^2 m_2/\sqrt{q^2} \right) + \dots \right], \\
H_{--} &= 8N_c q^2 \left[ 0 + \frac{\alpha_s}{6\pi} \left( 1 + (\pi^2 - 16)m_2/\sqrt{q^2} \right) + \dots \right], \tag{2}
\end{aligned}$$

(cf. [1]) where we have expanded the result up to  $O(m_2/\sqrt{q^2})$ . Surprisingly the mass corrections to the NLO terms are linear in the antiquark mass and carry rather large coefficients.<sup>2</sup>

From (1) one can define a normalized decay distribution  $\widehat{W}(\theta)$  by replacing the density matrix elements and the polarized decay functions in (1) by their normalized forms  $\hat{\rho}_{mm} = \rho_{mm}/\sum_m \rho_{mm}$  and  $\hat{H}_{mm} = H_{mm}/\sum_m H_{mm}$ . One then has  $\int d\cos\theta \widehat{W}(\theta) = 1$ .

The angular decay distribution (1) and its normalized form  $\widehat{W}(\theta)$  are second-order polynomials in  $\cos\theta$  and therefore have the functional form of a parabola. At threshold  $q^2 = (m_1 + m_2)^2$  one has  $H_{++} = H_{00} = H_{--}$  and the angular decay distribution becomes flat with  $\widehat{W}(\theta) = 1/2$ .

The normalized decay distribution  $\widehat{W}(\theta)$  can be characterized by the convexity parameter (see e.g. [1, 2])

$$c_f = \frac{d^2 \widehat{W}(\theta)}{d(\cos\theta)^2} = \frac{3}{4} (\hat{\rho}_{++} - 2\hat{\rho}_{00} + \hat{\rho}_{--})(\hat{H}_{++} - 2\hat{H}_{00} + \hat{H}_{--}). \tag{3}$$

When  $c_f$  is negative (positive) the angular decay distribution is described by a downward (upward) open parabola. The convexity parameter vanishes for the two cases (i) unpolarized gauge boson where  $\hat{\rho}_{++} = \hat{\rho}_{00} = \hat{\rho}_{--}$  and/or (ii) zero analyzing power where  $\hat{H}_{++} = \hat{H}_{00} = \hat{H}_{--}$ .

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<sup>2</sup>For  $Z$  decays one has to replace the  $H_{mm}$  by  $(v_f^2 H_{mm}^{VV} + a_f^2 H_{mm}^{AA})$  etc. in (1), where  $v_f$  and  $a_f$  are the weak coupling coefficients of the SM.

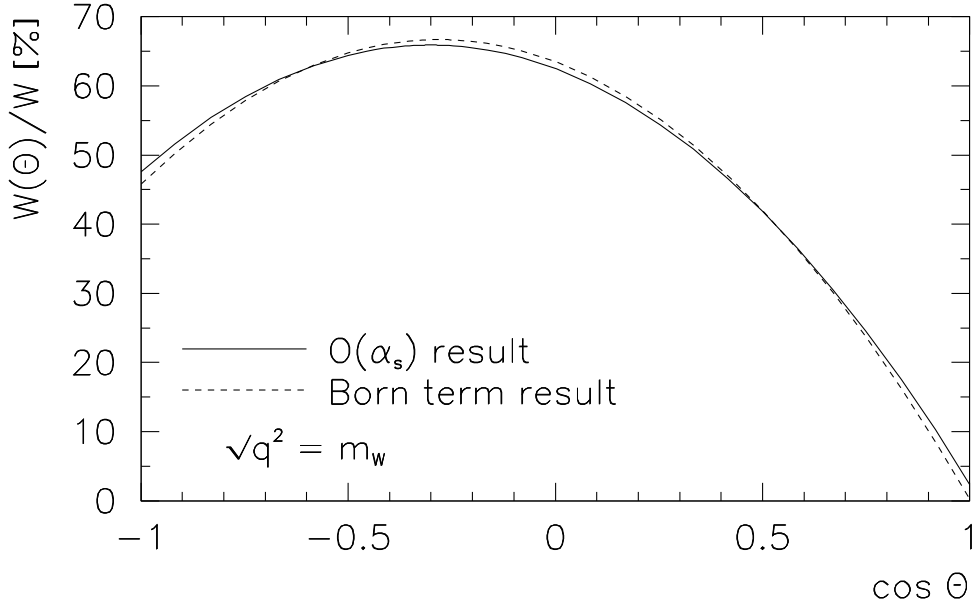


Figure 1: Normalized angular decay distribution  $\widehat{W}(\theta) = W(\theta)/W$  in the decay  $W^+(\uparrow) \rightarrow c\bar{b}$  at LO (dashed line) and NLO (full line). NLO result contains both initial-state and final-state corrections.

### 3 The sequential decay $t \rightarrow b + W^+(\rightarrow c\bar{b})$

At LO the normalized density matrix elements of the  $W^+$  in the decay  $t \rightarrow b + W^+$  are given by  $\hat{\rho}_{++} = 0$ ,  $\hat{\rho}_{00} = (1 + 2x^2)^{-1}$  and  $\hat{\rho}_{--} = 2x^2/(1 + 2x^2)^{-1}$ , where  $x = m_W/m_t$  [3]. The  $z$  and  $z'$  axes are defined by the momentum of the  $W^+$  in the top rest frame and the momentum of the  $\bar{b}$ -antiquark. NLO and NNLO corrections to the density matrix elements  $\rho_{mm}$  have been calculated in [4] and in [5], respectively. In Fig. 1 we plot the normalized angular decay distribution for  $W^+(\uparrow) \rightarrow c\bar{b}$ . It is apparent that the distribution becomes flatter when radiative corrections are applied. Numerically one has  $c_f = -0.81$  and  $c_f = -0.75$  without and with radiative corrections.

### 4 The sequential decays

$$H \rightarrow W^- + W^{*+}(\rightarrow q_1\bar{q}_2) \text{ and } H \rightarrow Z + Z^*(\rightarrow q\bar{q})$$

Quark mass effects are much more important for the off-shell decays since the scale is not set by  $m_{W,Z}^2$  but by the off-shellness  $q^2$  which varies in the range  $(m_1 + m_2)^2 \leq q^2 \leq (m_H - m_{W,Z})^2$ . There are also new scalar contributions to the decay well familiar from neutron  $\beta$  decay or e.g. from the decay  $B \rightarrow (D, D^*) + \tau\nu_\tau$  where the scalar contributions have been calculated to amount to (67%, 37%) of the total decay rate [6]. The scalar contributions can be isolated by splitting the off-shell propagator of the  $W^*, Z^*$  into a

spin-1 and a spin-0 piece by writing<sup>3</sup>

$$\left(-g^{\mu\mu'} + \frac{q^\mu q^{\mu'}}{m_{W,Z}^2}\right) = \left(-g^{\mu\mu'} + \frac{q^\mu q^{\mu'}}{q^2} - \frac{q^\mu q^{\mu'}}{q^2}\left(1 - \frac{q^2}{m_{W,Z}^2}\right)\right). \quad (4)$$

The angular decay distribution (1) is now augmented by a scalar contribution leading to

$$W_{\text{off-shell}}(\theta) = \frac{3}{2} \sum_{m,m'=0,\pm} \rho_{mm} d_{mm'}^1(\theta) d_{mm'}^1(\theta) H_{m'm'} - \frac{3}{2} \left(1 - \frac{q^2}{m_W^2}\right) (\rho_{t0} H_{t0} + \rho_{0t} H_{0t}) \cos \theta + \frac{3}{2} \left(1 - \frac{q^2}{m_W^2}\right)^2 \rho_{tt} H_{tt}. \quad (5)$$

The scalar analyzing functions  $H_{t0}$ ,  $H_{0t}$  and  $H_{tt}$  are proportional to the square of the quark masses and, therefore, the angular distribution (5) collapses back into the form (1) in the zero quark mass limit.

In the SM the Higgs couples to the gauge bosons via the metric tensor. The resulting density matrix elements of the  $W^*$ ,  $Z^*$  bosons in the decay  $H \rightarrow WW^*, ZZ^*$  have been calculated in [2].

Numerical results for off-shell and quark mass effects in the decay  $H \rightarrow W^- + W^{*+} (\rightarrow c\bar{b})$  can be found in [2]. Here we concentrate on numerical results for the decay  $H \rightarrow Z + Z^* (\rightarrow b\bar{b})$ . At LO one finds that the scalar contribution not present in the zero quark mass limit amounts to 8.6 % of the total decay rate. In the vicinity of the threshold region the LO convexity parameter is given by

$$c_f = -\frac{3}{2} \frac{q^2 - 4m_b^2}{q^2 + 2m_b^2}. \quad (6)$$

The convexity parameter vanishes at threshold  $q^2 = 4m_b^2$  as it must, whereas one has  $c_f = -3/2$  at  $q^2 = 0$  in the zero mass limit. One anticipates big differences of the angular decay distribution in the vicinity of the threshold for the two cases. This shows up in the angular decay distribution Fig. 2 where we plot the angular decay distribution for  $q^2 = 150 \text{ GeV}^2$  which is sufficiently far above the nonperturbative regime of the  $(b\bar{b})$  channel. The angular decay distribution is much flatter for the massive case. Fig. 2 also includes a plot of the NLO radiative corrections to the massive case which can be seen to be quite small.

**Acknowledgements:** This work was supported by the Estonian target financed project No. 0180056s09, and by the Estonian Science Foundation under grant No. 8769. S.G. acknowledges support by the Forschungszentrum of the Johannes-Gutenberg-Universität Mainz “Elementarkräfte und Mathematische Grundlagen (EMG)”.

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<sup>3</sup>In low energy applications the term  $q^2/M_{W,Z}^2$  in the scalar part is usually dropped.

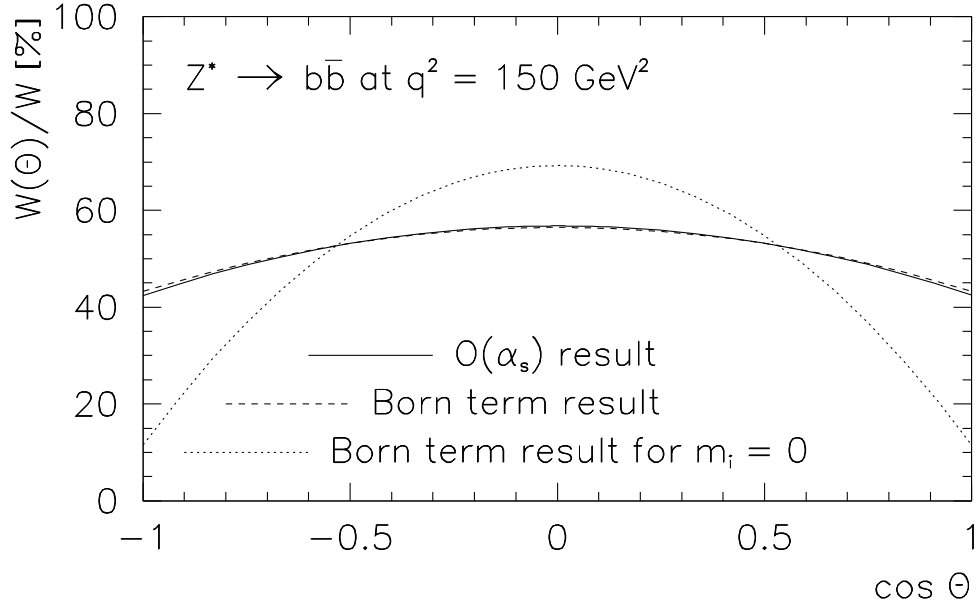


Figure 2: Polar angle distribution for  $Z^*(\rightarrow b + \bar{b})$  at  $q^2 = 150 \text{ GeV}^2$ . The three curves correspond to (i) Born term ( $m_b = 0$ ) (dotted line) (ii) Born term ( $m_b \neq 0$ ) (dashed line) and (iii)  $O(\alpha_s)$  with ( $m_b \neq 0$ ) (full line).

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